

# CMPS 112: Spring 2019

## Comparative Programming Languages

### *Course review*

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Based on course materials developed by Ranjit Jhala

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## The Lambda Calculus

- Lambda calculus terms
  - variables, abstractions, & applications
- Variable scope
  - Free vs bound variables
- Evaluation
  - Alpha renaming
  - Beta reduction
  - Normal form
- Church encodings
  - numbers, booleans, etc
- Recursion
  - Fixed-point combinator

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## Haskell

- A typed, lazy, purely functional programming language
  - Haskell =  $\lambda$ -calculus +
    - Better syntax
    - Types
    - Built-in features
      - Booleans, numbers, characters
      - Records (tuples)
      - Lists
      - Recursion
      - ...

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## Midterm review

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## Haskell topics

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- Haskell's type system
  - Recognizing / understanding relationship between Haskell expressions and their types
- Algebraic data types
  - Records
  - Sum types
  - Recursive ADTs
- Pattern matching
  - Overlapped / missing patterns
- Writing algorithms on (recursive) ADTs
  - Base cases + inductive cases

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## Higher Order Functions

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Iteration patterns over collections:

- **Filter** values in a collection given a *predicate*
- **Map** (iterate) a given *transformation* over a collection
- **Fold** (reduce) a collection into a value, given a *binary operation* to combine results

Useful helper HOFs:

- **Flip** the order of function's (first two) arguments
- **Compose** two functions

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# Evaluating Nano1

Back to our expressions... now with environments!

```
data Expr = Num Int           -- number
          | Var Id             -- variable
          | Bin Binop Expr Expr -- binary expression
          | Let Id Expr Expr   -- let expression
```

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# Static vs Dynamic Scoping

Dynamic scoping:

- each occurrence of a variable refers to the most recent binding *during program execution*
- can't tell where a variable is defined just by looking at the function body
- nightmare for readability and debugging:

```
let cTimes = \x -> c * x in
let c = 5 in
let res1 = cTimes 2 in -- ==> 10
let c = 10 in
let res2 = cTimes 2 in -- ==> 20!!!
res2 - res1
```

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# Static vs Dynamic Scoping

What we want:

```
let c = 42 in
let cTimes = \x -> c * x in
let c = 5 in
cTimes 2
=> 84
```

Lexical (or static) scoping:

- each occurrence of a variable refers to the most recent binding *in the program text*
- definition of each variable is unique and known *statically*
- good for readability and debugging: don't have to figure out where a variable got "assigned"

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## Static vs Dynamic Scoping

What we don't want:

```
let c = 42 in
let cTimes = \x -> c * x in
let c = 5 in
cTimes 2
=> 10
```

Dynamic scoping:

- each occurrence of a variable refers to the most recent binding *during program execution*
- can't tell where a variable is defined just by looking at the function body
- nightmare for readability and debugging:

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## Static vs Dynamic Scoping

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res2 - res1
```

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## Closures

To implement lexical scoping, we will represent function values as *closures*

Closure = *lambda abstraction* (formal + body) + *environment* at function definition

```
data Value = VNum Int
           | VClos Env Id Expr -- env + formal + body
```

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# Grammars

A grammar is a recursive definition of a set of trees

- each tree is a *parse tree* for some string
- *parse* a string  $s$  = find a parse tree for  $s$  that belongs to the grammar

A grammar is made of:

- **Terminals:** the leaves of the tree (tokens!)
- **Nonterminals:** the internal nodes of the tree
- **Production Rules** that describe how to “produce” a non-terminal from terminals and other non-terminals
  - i.e. what children each nonterminal can have:

```
Aexpr : -- NT Aexpr can have as children:  
| Aexpr '+' Aexpr { ... } -- NT Aexpr, T '+', and NT Aexpr, or  
| Aexpr '-' AExpr { ... } -- NT Aexpr, T '-', and NT Aexpr, or  
| ...
```

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# Type system for Nano2

A **type system** defines what types an expression can have

To define a type system we need to define:

- the *syntax* of types: what do types look like?
- the *static semantics* of our language (i.e. the typing rules): assign types to expressions

```
G |- e :: T
```

An expression  $e$  has **type  $T$**  in  $G$  if we can derive  $G \mid - e :: T$  using these rules

An expression  $e$  is **well-typed** in  $G$  if we can derive  $G \mid - e :: T$  for some type  $T$

- and **ill-typed** otherwise

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# Double identity

```
let id = \x -> x in  
  let y = id 5 in  
    id (\z -> z + y)
```

Intuitively this program looks okay, but our type system *rejects* it:

- in the first application,  $id$  needs to have type  $Int \rightarrow Int$
- in the second application,  $id$  needs to have type  $(Int \rightarrow Int) \rightarrow (Int \rightarrow Int)$
- the type system forces us to pick *just one type* for each variable, such as  $id : ($

What can we do?

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## Inference with polymorphic types

With polymorphic types, we can derive  $e :: \text{Int} \rightarrow \text{Int}$  where  $e$  is

```
let id = \x -> x in
  let y = id 5 in
    id (\z -> z + y)
```

At a high level, inference works as follows:

1. When we have to pick a type  $T$  for  $x$ , we pick a **fresh type variable**  $a$
2. So the type of  $\backslash x \rightarrow x$  comes out as  $a \rightarrow a$
3. We can **generalize** this type to  $\text{forall } a . a \rightarrow a$
4. When we apply  $\text{id}$  the first time, we **instantiate** this polymorphic type with  $\text{Int}$
5. When we apply  $\text{id}$  the second time, we **instantiate** this polymorphic type with  $\text{Int} \rightarrow \text{Int}$

Let's formalize this intuition as a type system!

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## Typing rules

We need to change the typing rules so that:

1. Variables (and their definitions) can have polymorphic types

```
[T-Var] G |- x :: S          if x:S in G
```

```
      G |- e1 :: S   G, x:S |- e2 :: T
[T-Let] -----
      G |- let x = e1 in e2 :: T
```

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## Typing rules

2. We can *instantiate* a type scheme into a type

```
      G |- e :: forall a . S
[T-Inst] -----
      G |- e :: [a / T] S
```

3. We can *generalize* a type with free type variables into a type scheme

```
      G |- e :: S
[T-Gen] ----- if not (a in FTV(G))
      G |- e :: forall a . S
```

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## Typing rules

The rest of the rules are the same:

[T-Num]  $G \vdash n :: \text{Int}$

[T-Add] 
$$\frac{G \vdash e_1 :: \text{Int} \quad G \vdash e_2 :: \text{Int}}{G \vdash e_1 + e_2 :: \text{Int}}$$

[T-Abs] 
$$\frac{G, x:T_1 \vdash e :: T_2}{G \vdash \lambda x \rightarrow e :: T_1 \rightarrow T_2}$$

[T-App] 
$$\frac{G \vdash e_1 :: T_1 \rightarrow T_2 \quad G \vdash e_2 :: T_1}{G \vdash e_1 e_2 :: T_2}$$

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## Formalizing Nano

Goal: we want to guarantee properties about programs, such as:

- evaluation is deterministic
- all programs terminate
- certain programs never fail at run time
- etc.

To prove theorems about programs we first need to define formally

- their *syntax* (what programs look like)
- their *semantics* (what it means to run a program)

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## Nano1: Operational Semantics

We define the step relation *inductively* through a set of *rules*:

[Add-L] 
$$\frac{e_1 \Rightarrow e_1' \quad \text{-- premise}}{e_1 + e_2 \Rightarrow e_1' + e_2 \quad \text{-- conclusion}}$$

[Add-R] 
$$\frac{e_2 \Rightarrow e_2'}{n_1 + e_2 \Rightarrow n_1 + e_2'}$$

[Add]  $n_1 + n_2 \Rightarrow n \quad \text{where } n == n_1 + n_2$

[Let-Def] 
$$\frac{e_1 \Rightarrow e_1'}{\text{let } x = e_1 \text{ in } e_2 \Rightarrow \text{let } x = e_1' \text{ in } e_2}$$

[Let]  $\text{let } x = v \text{ in } e_2 \Rightarrow e_2[x := v]$

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## Operational semantics

We need to extend our reduction relation with rules for abstraction and application:

$$\begin{array}{c} e1 \Rightarrow e1' \\ \text{[App-L]} \text{-----} \\ e1 e2 \Rightarrow e1' e2 \end{array}$$
$$\begin{array}{c} e \Rightarrow e' \\ \text{[App-R]} \text{-----} \\ v e \Rightarrow v e' \end{array}$$
$$\text{[App]} \quad (\lambda x \rightarrow e) v \Rightarrow e[x := v]$$

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## Now what?

Did you like what you learned here? Want to learn more?

- **CSE 114 (not 116) Functional Programming**
  - Winter 2020, Cormac Flanagan
- **CSE 110A Fundamentals of Compiler Design**
  - Fall 2019, Spring 2020, Wesley Mackey
  - Winter 2020, me
- **CSE 210A: Programming languages**
  - Winter 2020, TBD
- **CSE 210B: Adv. Programming languages**
  - Winter 2020, Cormac Flanagan
  - Spring 2020, me

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## Thanks and good luck!

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